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Method of Basic Projectiles for Calculating Force Coefficients

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Nomenclature

$\boldsymbol{a}$ = vector of global parameters characterizing body–stream interaction
$c_{A\nu}$ = axial force coefficient
$c_{D\nu}$ = drag coefficient
$c_{L\nu}$ = lift coefficient
$c_{N\nu}$ = normal force coefficient
c_ν = projection of the aerodynamic forces coefficients on the direction ℓ°
L_ν = length of a projectile, m
$\boldsymbol{\ell}^\circ$ = vector determining a direction; see Fig. 1
M_∞ = freestream Mach number
$\boldsymbol{n}^\circ$ = local inner normal vector at the projectile surface
R_ν = characteristic dimension of a base of a projectile, m; see Fig. 1
s_ν = cross-sectional area of a projectile, m^2
t_ν = power in equation of longitudinal contour of a projectile
U_ν = current value of $[\mathrm{d}\phi_\nu/\mathrm{d}x_\nu]^{-1}$
$\underline{U}_\nu, \bar{U}_\nu$ = values of U_ν at boundary points of the longitudinal contour
$\boldsymbol{v}_\infty$ = freestream velocity, m/s
x_ν, y_ν, z_ν = coordinates; see Fig. 1
α = angle of attack
$\eta(\theta_\nu)$ = function characterizing cross-sectional contour of a projectile
κ = angle between $\boldsymbol{\ell}^\circ$ and axis x_ν; see Fig. 1
λ_ν = aspect ratio
ρ_ν, θ_ν = coordinates; see Fig. 1
$\boldsymbol{\tau}^\circ$ = local surface point tangent vector, being in plane of vectors $\boldsymbol{n}^\circ$ and $\boldsymbol{v}^\circ_\infty$
$\phi_\nu(x_\nu)$ = increasing function determining the longitudinal contour of a projectile
Ω_p, Ω_τ = functions determining the specific model of a projectile–fluid interaction
ω = angle between vectors $\boldsymbol{n}^\circ$ and $\boldsymbol{v}^0_\infty$

Subscripts

ν = number of a projectile
$*$ = projectile with the unknown aerodynamic characteristics

Superscripts

$\dot{\phi}_\nu$ = derivative with respect to x_ν
0 = unit vector
$'$ = derivative with respect to U

Introduction

MODERN supersonic/hypersonic gasdynamics employs localized interaction models that determine projectile–fluid interaction at any location on the projectile as a function of a local inclination of its surface to the freestream flow and global parameters, e.g., Mach and Reynolds numbers. Various specific models of this type for free-molecular, intermediate, and continuum flow regimes are described in Ref. 1. The calculation of the total force coefficients is performed via integration of the local coefficients over the surface of a projectile or using some specially developed procedures.[1–4] Because usually localized interaction models are approximate, there are uncertainties concerning the choice of a particular model or its parameters for a given problem, even among the most widely used models.[5]

The essence of the method developed in this study is that it implies the local character of projectile–fluid interaction but does not require specification of the model and its parameters. Aerodynamic characteristics of a projectile are found by recalculation of the known characteristics for several basic projectiles that are determined either experimentally or with an exact model for the same flight conditions. The localized interaction model for all of these projectiles is considered to be the same although unknown.

Formulation of the Problem

Let us consider n basic projectiles $T_1, T_2, \ldots, T_n$ with convex surfaces moving in a fluid under the conditions of the localized projectile–medium interaction; i.e., the local force coefficient can be represented in the following form:

$$\boldsymbol{C}_F = \Omega_p(\boldsymbol{a}, \omega)\boldsymbol{n}^\circ + \Omega_\tau(\boldsymbol{a}, \omega)\boldsymbol{\tau}^\circ, \qquad \boldsymbol{a} = \{a_1, a_2, \ldots\} \tag{1}$$

All cross sections of all projectiles are geometrically similar to some convex figure, e.g., circle and ellipse. The notations for the geometry are presented in Fig. 1. All of the sizes are given in dimensionless units, whereby the values R_ν and s_ν denote a characteristic dimension and a characteristic area for the νth projectile, respectively. The equation of the surface of such projectile T_ν can be written as

$$\rho_\nu = \phi_\nu(x_\nu)\cdot\eta(\theta_\nu), \qquad 0 \le \theta_\nu \le 2\pi, \qquad 0 \le x_\nu \le L_\nu/R_\nu \tag{2}$$

$$\eta(0) = \eta(2\pi) = 1, \qquad \phi_\nu(0) = 0, \qquad \phi_\nu(2\lambda_\nu) = 1 \tag{3}$$

It is assumed that

$$\bigcup_{\nu=1}^{n}[\underline{U}_\nu, \bar{U}_\nu] \stackrel{\text{def}}{=} [\underline{U}_0, \bar{U}_0] \ne \varnothing \tag{4}$$

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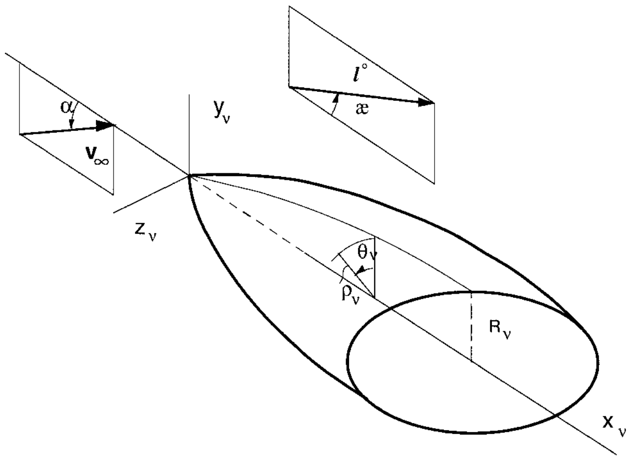


Fig. 1 Shape of the projectile and notations.

where

$$\underline{U}_\nu = \frac{1}{\dot{\phi}_\nu(0)}, \qquad \bar{U}_\nu = \frac{1}{\dot{\phi}_\nu(2\lambda_\nu)} \tag{5}$$

i.e., the union of intervals $[\underline{U}_\nu, \bar{U}_\nu]$ for all projectiles covers some interval $[\underline{U}_0, \bar{U}_0]$ with possible overlappings but without "holes." In the particular case[1] $\underline{U}_\nu = \underline{U}_0, \bar{U}_\nu = \bar{U}_0, \nu = 1, \ldots, n$, the tangents at the longitudinal contours of basic and corresponding projectiles must be the same at the initial and final points, which essentially restricts the application of the method.

The component of integral aerodynamic force as its projection in any direction ℓ° (see Fig. 1) reads

$$c_\nu(\kappa, \alpha) = \frac{1}{s_\nu}\ell^\circ \iint \boldsymbol{C}_{F\nu}\,\mathrm{d}S = \int_0^{2\lambda_\nu} \phi_\nu(x_\nu)\cdot H(\kappa, \alpha, \dot{\phi}_\nu)\,\mathrm{d}x_\nu \tag{6}$$

where integration is usually performed over the exposed region of the projectile's surface determined by the condition $\cos\omega \geq 0$, but if it is required, the effect of the flow on the shaded area may be taken into account as well. Equation (6) provides the expression for $c_{D\nu}, c_{L\nu}, c_{A\nu}$, and $c_{N\nu}$ if κ is set equal to $\alpha, \pi/2+\alpha, 0$, and $\pi/2$, respectively. The expression for the function H is determined by the specific model and by the projectile's cross section, and it is not essential for the further analysis and is not presented here.

Our immediate goal is to prove that, for a given set $\beta_1, \beta_2, \ldots, \beta_n$, there exists a corresponding projectile T_0 with a surface determined by Eqs. (2) and (3) for $\nu = 0$ such that $\underline{U}_0 \leq [\dot{\phi}_0(x_0)]^{-1} \leq \bar{U}_0$, and the following invariant equation is satisfied:

$$c_0(\kappa, \alpha) = \sum_{\nu=1}^{n} \beta_\nu c_\nu(\kappa, \alpha) \tag{7}$$

which is valid for an arbitrary function H, i.e., for any specific localized interaction model, for arbitrary value of κ, i.e., for each component of the integral aerodynamic force, for arbitrary shape of a cross section and an arbitrary angle of attack, which are the same for all projectiles.

Invariant Relations

Define the following sequence ordered with respect to the magnitude of its terms:

$$\underline{U}_0 = W^{(0)} < W^{(1)} < \cdots < W^{(m-1)} < W^{(m)} = \bar{U}_0 \tag{8}$$

where $W^{(i)}$ denotes $\underline{U}_\nu$ or $\bar{U}_\nu$ and equal terms are considered as a single term. Then Eq. (6) can be rewritten as

$$c_\nu(\kappa, \alpha) = \sum_{i=\underline{k}_\nu}^{\bar{k}_\nu} \int_{W^{(i-1)}}^{W^{(i)}} \hat{\phi}_\nu(U)\cdot\hat{x}'_\nu(U)\cdot H(\kappa, \alpha, U^{-1})\,\mathrm{d}U \tag{9}$$

where the parametric representation of the function $\phi_\nu = \phi_\nu(x_\nu)$ is used:

$$x_\nu = \hat{x}_\nu(U), \qquad \phi_\nu = \hat{\phi}_\nu(U), \qquad U = \frac{1}{\dot{\phi}_\nu(x_\nu)} \tag{10}$$

and $\underline{k}_\nu$ and $\bar{k}_\nu$ are determined by the conditions $W^{(\underline{k}_\nu)} = \underline{U}_\nu$ and $W^{(\bar{k}_\nu)} = \bar{U}_\nu$. Equations (2) and (10) imply that

$$\hat{x}'_\nu(U) = U\cdot\hat{\phi}'_\nu(U) \tag{11}$$

$$\hat{x}_\nu(\underline{U}_\nu) = 0, \qquad \hat{\phi}_\nu(\underline{U}_\nu) = 0$$
$$\hat{x}_\nu(\bar{U}_\nu) = 2\lambda_\nu, \qquad \hat{\phi}_\nu(\bar{U}_\nu) = 1 \tag{12}$$

Function $\phi_0(x_0)$ will be constructed in a parametric form of Eq. (10) with $\nu = 0$ as a set of functions $\hat{x}_0^{(i)}(U), \hat{\phi}_0^{(i)}(U)$ defined at the intervals $[W^{(i-1)}, W^{(i)}]$ $(i = 1, 2, \ldots, m)$ and taking into account the conditions at the initial point of a contour and continuity conditions at the ends of the adjacent intervals:

$$g^{(1)}\left[W^{(0)}\right] = 0, \qquad g^{(i)}\left[W^{(i)}\right] = g^{(i+1)}\left[W^{(i)}\right]$$
$$g = \hat{x}_0, \hat{\phi}_0, \qquad i = 1, 2, \ldots, m-1 \tag{13}$$

which will ensure the continuity of function $\phi_0(x_0)$ and its derivative. Equation (6) for the aerodynamic characteristics of a corresponding projectile can be written as

$$c_0(\kappa, \alpha) = \sum_{i=1}^{m} \int_{W^{(i-1)}}^{W^{(i)}} \hat{\phi}_0^{(i)}(U)\cdot\hat{x}_0^{(i)\prime}(U) H(\kappa, \alpha, U^{-1})\,\mathrm{d}U \tag{14}$$

The required set of functions determining the corresponding projectile is as follows:

$$\hat{\phi}_0^{(i)} = \sqrt{\xi^{(i)}(U)}, \qquad \xi^{(i)}(U) = \sum_{\substack{\nu \\ (\underline{k}_\nu \leq i)\wedge(\bar{k}_\nu \geq i)}} \beta_\nu \hat{\phi}_\nu^2(U) + A^{(i)} \tag{15}$$

$$\hat{x}_0^{(i)} = U\hat{\phi}_0^{(i)}(U) - \int_{W^{(i-1)}}^{U} \hat{\phi}_0^{(i)}(t)\,\mathrm{d}t - B^{(i)} \tag{16}$$

where

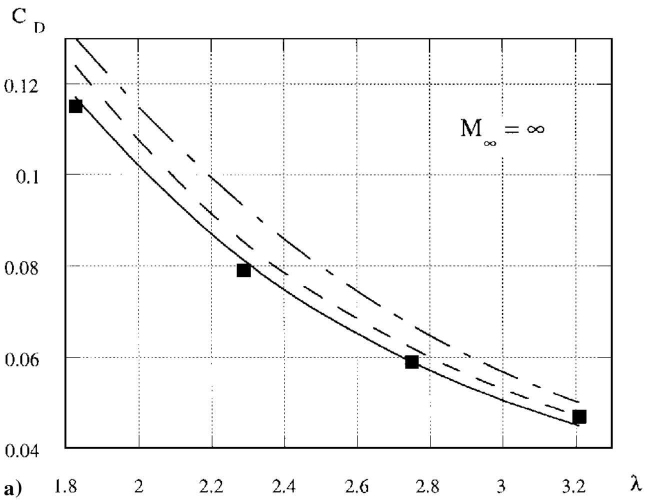
$$A^{(1)} = B^{(1)} = 0, \qquad A^{(i)} = \sum_{\bar{k}_\nu < i} \beta_\nu$$
$$B^{(i)} = \sum_{j=2}^{i} \int_{W^{(j-2)}}^{W^{(j-1)}} \hat{\phi}^{(j-1)}(t)\,\mathrm{d}t, \qquad i = 2, 3, \ldots, m \tag{17}$$

These functions satisfy the boundary conditions given by Eqs. (17) and (7). The latter can be verified by substituting these functions into the integral in Eq. (14).

Equation (15) implies that the choice of the parameters β_ν is limited by the conditions $\xi^{(i)}(U) \geq 0$, $i = 1, 2, \ldots, m$. It can be shown that the longitudinal contour of a projectile T_0 is convex if $\xi^{(i)\prime}(U) > 0, i = 1, 2, \ldots, m$. Both of these conditions are satisfied if all $\beta_\nu \geq 0$. Because we used the dimensionless variables, the condition

$$\sum_{\nu=1}^{n} \beta_\nu = 1$$

must be added to provide $\hat{\phi}_0(\bar{U}_0) = 1$.



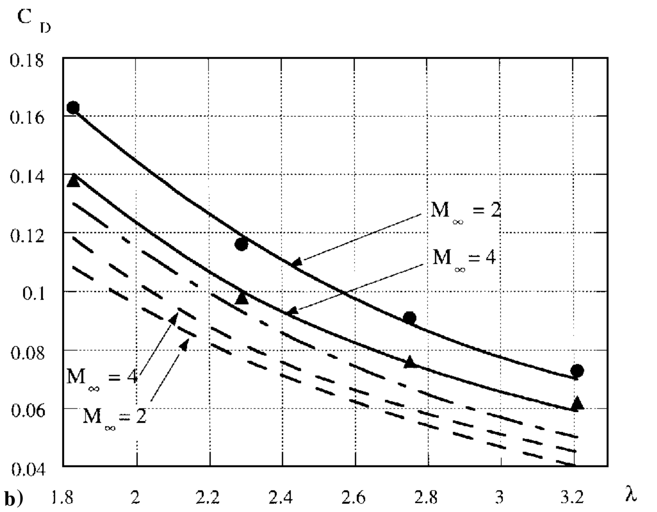


Fig. 2 Comparison of various methods: ——, exact results; — -, the Newtonian model; – – –, the modified model; and symbols, method of basic projectiles.

If all of the projectiles are blunt shaped, i.e., $\bar{U}_\nu = W^{(0)} = 0$, $\nu = 1, 2, \ldots, n$, then the basic projectiles can be ordered so that $\bar{U}_1 < \bar{U}_2 < \cdots < \bar{U}_n = \bar{U}_0$ and the preceding formulas may be simplified. In particular, Eq. (15) can be written as

$$\hat{\phi}_0^{(i)}(U) = \left[\sum_{\nu=i}^{n} \beta_\nu \hat{\phi}_\nu^2(U) + \sum_{\nu=1}^{i-1} \beta_\nu \right]^{\frac{1}{2}} \tag{18}$$

while the second term is set equal to zero if $i = 1$.

Applications to Aerodynamic Calculations

Assume that values of c_ν for several basic projectiles are known, for example, from experiment. Then, varying values of β_ν enables us to construct a set of the corresponding projectiles T_0 (β_1, $\beta_2, \ldots, \beta_n$). The shape of these projectiles is determined from Eqs. (15–17), and their aerodynamic characteristics can be calculated from Eq. (7). Invariance of Eq. (7) yields a useful property, namely, that their variation under conditions $\beta_\nu \geq 0$ yields an estimate:

$$\min_{1 \leq \nu \leq n} c_\nu(\kappa, \alpha) \leq c_0(\kappa, \alpha) \leq \max_{1 \leq \nu \leq n} c_\nu(\kappa, \alpha) \tag{19}$$

The latter condition implies that, in the hypersonic free-molecular, intermediate, and continuum flight conditions, when the localized interaction models are valid, the values of aerodynamic characteristics of the corresponding projectiles for each flight regime lie in the range between the minimum and maximum values of these characteristics for the basic projectiles under the same angle of attack. Varying β_ν allows us to change the shape of a corresponding projectile while keeping its characteristics in a given interval.

If the shape of a projectile T_* with the unknown aerodynamic characteristics is given, then the problem of determining its aerodynamic characteristics, generally, has no exact solution. However, among the corresponding projectiles $T_0(\beta_1, \beta_2, \ldots, \beta_n)$, one can find a projectile with a shape close to the shape of the projectile T_* with the aerodynamic characteristics that can be accepted as an estimate for the desired aerodynamic characteristics.

Here we present analysis of calculations for power law bodies of revolution with the shape in supersonic and hypersonic flows of a perfect gas. Results of exact calculations,[6] which were validated by comparison with the experiments, were used for evaluation of the performance of our method for calculating drag coefficient for $\alpha = 0$. In the calculations, we used the following values of parameters for the first and second projectiles: $t_1 = 0.5$, $\lambda_1 = 1.5$, and $t_2 = 0.6$; the values of λ_2 for the second basic projectile are 2.0, 2.5, 3.0, and 3.5. The drag coefficient of the corresponding projectile with $t_* = 0.55$ was determined for pairs of the basic projectiles. The aspect ratio λ_* was found from condition $\bar{U}_* = \bar{U}_2$, and the method of least squares was used to calculate values β_ν from Eq. (18).

Dependencies of calculated values of drag coefficient vs parameter λ for $t_* = 0.55$ are shown in Fig. 2. For comparison, in Fig. 2 we present the results of calculations using the model with $\Omega_p = a_1 \cos^2 \omega$, $\Omega_\tau = 0$, where $a_1 = 2$ for the Newtonian model and a_1 was calculated using the Rayleigh formula for the modified model. Results presented in Fig. 2 show that values of drag coefficient obtained using the invariant relations are close to the exact ones in a wide range of Mach numbers. At $M_\infty = \infty$ (Fig. 2a), the modified model and method of invariant relations produce results that are close to the exact ones, whereas the results obtained with the Newtonian model have higher error. At moderate supersonic velocities (Fig. 2b), the accuracy of the method of basic projectiles remains essentially the same as at $M_\infty = \infty$, whereas the errors of the modified models and the Newtonian model are rather large. The latter demonstrates that in this case an approach based on a general assumption of a localized nature of projectile–fluid interaction proved to be more accurate than specific models.

Concluding Remark

The proposed method is intended primarily for use in applied aerodynamics at the draft stage of vehicle design. It is desirable to compile the database containing information about the aerodynamic characteristics of different projectiles for different flight conditions to provide systematic practical calculations. This could enable us to select basic projectiles automatically from the database for each specific calculation.

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